

March 2001

AdS/CFT and quantum-corrected brane entropy

SHIN'ICHI NOJIRI¹, and SERGEI D. ODINTSOV^{♠2}

*Department of Applied Physics
National Defence Academy, Hashirimizu Yokosuka 239-8686, JAPAN*

*♠ Instituto de Fisica de la Universidad de Guanajuato, Lomas del Bosque
103, Apdo. Postal E-143, 37150 Leon, Gto., MEXICO and Tomsk State
Pedagogical University, 634041 Tomsk, RUSSIA*

ABSTRACT

It is shown that quantum-induced (inflationary) brane Universe occurs in the bulk 5d AdS black hole in accordance with AdS/CFT correspondence. Brane stress tensor is induced by quantum effects of dual CFT and brane crosses the horizon of AdS black hole. Quantum-corrected Hubble constant, Hawking temperature and entropy are found on the brane (and at the horizon). The similarity between CFT entropy at the horizon and FRW equations is extended on the quantum level. This suggests the way to understand cosmological entropy bounds in quantum gravity.

¹nojiri@cc.nda.ac.jp

² odintsov@ifug5.ugto.mx, odintsov@mail.tomsknet.ru

1 Introduction

It is quite well-known fact that holographic principle suggests the interesting bounds between microscopic and Bekenstein-Hawking entropy [1] as it was discussed in refs.[2, 3]. Recently, the very interesting attempt to study the holographic principle in Friedmann-Robertson-Walker (FRW) Universe filled by conformal matter has been done by Verlinde [4]. Using dual AdS-description [5] it has been found the relation between entropy (energy) of CFT and cosmological equations controlling the behaviour of scale factor in FRW Universe. In particular, the equation controlling the entropy bounds during evolution has been obtained [4] and Cardy-Verlinde formula has been derived. These results have been subsequently generalized and discussed in a number of works [6, 7].

From another side there is some interest to the cosmological brane universe realized as some kind of the boundary in the AdS-Schwarzschild black hole as it was discussed in [8]. Related with the above works about the holographic principle in FRW universe, one interesting extension has been presented in ref.[9] where similar questions about the cosmological entropy, evolution, etc. within the holographic principle have been studied from classical brane-world perspective[10]. In particular, the behaviour of the CFT entropy at the horizon of bulk 5d AdS BH has been investigated and its comparison with FRW equations has been done.

In the study of brane-worlds and their applications two main approaches could be considered. In the first, more traditional approach one starts from the higher dimensional theory which gives the higher dimensional bulk solution (say, AdS space). The next step is to get the necessary brane universe. In order to achieve this one adds by hands some boundary terms (brane vacuum energy). In this way, almost any brane universe may be easily obtained.

There is, however, another way which is closely connected with AdS/CFT correspondence and quantum properties of the system under discussion. In this, second approach the bulk action is not modified. However, the boundary terms are not fine-tuned, they are predicted by some reasonable assumptions. First of all, part of surface terms represents the Gibbons-Hawking term which is responsible for getting the variational procedure to be well-defined. Second contribution to surface terms comes from the principle that leading divergence of bulk space (say, of AdS) should be cancelled. Final part is the dynamical one: it is produced by quantum effects of CFT on the

brane. After having such action the brane universe comes as the solution of equations of motion. Definitely, very few brane universes naturally appear as a result of such dynamical solution of equations of motion. In this way, so-called Brane New World [11, 12] has been constructed.

The purpose of the present paper is to generalize the situation described in ref.[9] to the case of above quantum-induced (or AdS/CFT induced) brane-worlds suggested in refs.[11, 12], where the quantum creation of the brane universe is discussed. In this way, from one side one gets quantum-corrected FRW Universe equations as they look from the point of view of not only brane observer but also from the point of view of quantum induced brane-world. From another side, one gets the quantum-corrected brane entropy as well as Hubble constant and Hawking temperature at the horizon. Finally, this may be considered as extension of scenario of refs.[11, 12] (see refs.[13] for related questions) which admits also generalization for the presence of non-trivial dilaton and (or) supersymmetrization [14] for the case when brane crosses the horizon of AdS-black hole.

2 Brane New World in AdS-Schwarzschild Black Hole

We assume the brane connects two bulk spaces and we may also identify the two bulk spaces as in [10] by imposing Z_2 symmetry. We start with the Minkowski signature action S which is the sum of the Einstein-Hilbert action S_{EH} with the cosmological term, the Gibbons-Hawking surface term S_{GH} , the surface counter term S_1 and the trace anomaly induced action \mathcal{W} :

$$S = S_{\text{EH}} + S_{\text{GH}} + 2S_1 + \mathcal{W}, \quad (1)$$

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^5x \sqrt{-g_{(5)}} \left(R_{(5)} + \frac{12}{l^2} \right), \quad (2)$$

$$S_{\text{GH}} = \frac{1}{8\pi G} \int d^4x \sqrt{-g_{(4)}} \nabla_\mu n^\mu, \quad (3)$$

$$S_1 = -\frac{6}{16\pi G l} \int d^4x \sqrt{g_{(4)}}, \quad (4)$$

$$\begin{aligned} \mathcal{W} = & b \int d^4x \sqrt{-\tilde{g}} \tilde{F} A + b' \int d^4x \sqrt{\tilde{g}} \left\{ A \left[2 \tilde{\square}^2 + \tilde{R}_{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu \right. \right. \\ & \left. \left. - \frac{4}{3} \tilde{R} \tilde{\square}^2 + \frac{2}{3} (\tilde{\nabla}^\mu \tilde{R}) \tilde{\nabla}_\mu \right] A + \left(\tilde{G} - \frac{2}{3} \tilde{\square} \tilde{R} \right) A \right\} \end{aligned} \quad (5)$$

$$-\frac{1}{12} \left\{ b'' + \frac{2}{3}(b + b') \right\} \int d^4x \sqrt{\tilde{g}} \left[\tilde{R} - 6 \tilde{\square} A - 6(\tilde{\nabla}_\mu A)(\tilde{\nabla}^\mu A) \right]^2$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices $_{(5)}$ and those in the boundary 4 dimensional spacetime are specified by $_{(4)}$ (for details, see [12]). In (3), n^μ is the unit vector normal to the boundary. The Gibbons-Hawking term S_{GH} is necessary in order to make the variational method well-defined when there is boundary in the spacetime. In (4), the coefficient of S_1 is determined from AdS/CFT [11]. The factor 2 in front of S_1 is coming from that we have two bulk regions which are connected with each other by the brane. In (5), one chooses the 4 dimensional boundary metric as $g_{(4)\mu\nu} = e^{2A} \tilde{g}_{\mu\nu}$, where $\tilde{g}_{\mu\nu}$ is a reference metric. G (\tilde{G}) and F (\tilde{F}) are the Gauss-Bonnet invariant and the square of the Weyl tensor. \mathcal{W} can be obtained by integrating the conformal anomaly with respect to the scale factor A of the metric tensor since the conformal anomaly should be given by the variation of the quantum effective action with respect to A . Note that quantum effects of brane CFT are taken into account via Eq.(5)³.

In the effective action (5) induced by brane quantum conformal matter, in general, with N scalar, $N_{1/2}$ spinor, N_1 vector fields, N_2 ($= 0$ or 1) gravitons and N_{HD} higher derivative conformal scalars, b , b' and b'' are [12]

$$b = \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\text{HD}}}{120(4\pi)^2}$$

$$b' = -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\text{HD}}}{360(4\pi)^2}, \quad b'' = 0. \quad (6)$$

For typical examples motivated by AdS/CFT correspondence one has:

a) $\mathcal{N} = 4$ $SU(N)$ SYM theory

$$b = -b' = \frac{N^2 - 1}{4(4\pi)^2}, \quad (7)$$

³In [15], the bulk gravitational Casimir effect has been considered and it has been found that the Casimir effect leads to deformation of 5d AdS space shape as well as of shape of branes. The account of bulk quantum effects, however, do not change the qualitative picture and the brane inflation still occurs. Then the role of bulk scalar quantum effect is not relevant in present context.

b) $\mathcal{N} = 2$ $Sp(N)$ theory

$$b = \frac{12N^2 + 18N - 2}{24(4\pi)^2}, \quad b' = -\frac{12N^2 + 12N - 1}{24(4\pi)^2}. \quad (8)$$

Note that b' is negative in the above cases. It is important to note that brane quantum gravity may be taken into account via the contribution to correspondent parameters b, b' .

Then on the brane, we have the following equation which generalizes the classical brane equation of the motion:

$$\begin{aligned} 0 = & \frac{48l^4}{16\pi G} \left(A_{,z} - \frac{1}{l} \right) e^{4A} + b' (4\partial_\tau^4 A + 16\partial_\tau^2 A) \\ & - 4(b + b') (\partial_\tau^4 A - 2\partial_\tau^2 A - 6(\partial_\tau A)^2 \partial_\tau^2 A). \end{aligned} \quad (9)$$

This equation is derived from the condition that the variation of the action on the brane, or the boundary of the bulk spacetime, vanishes under the variation over A . The first term proportional to $A_{,z}$ expresses the bulk gravity force acting on the brane and the term proportional to $\frac{1}{l}$ comes from the brane tension. The terms containing b or b' express the contribution from the conformal anomaly induced effective action (quantum effects). In (9), one uses the form of the metric as

$$ds^2 = dz^2 + e^{2A(z,\tau)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu, \quad \tilde{g}_{\mu\nu} dx^\mu dx^\nu \equiv l^2 (-d\tau^2 + d\Omega_3^2). \quad (10)$$

Here $d\Omega_3^2$ corresponds to the metric of 3 dimensional unit sphere.

As a bulk space, we consider 5d AdS-Schwarzschild black hole spacetime, whose metric is given by,

$$ds_{\text{AdS-S}}^2 = \frac{1}{h(a)} da^2 - h(a) dt^2 + a^2 d\Omega_3^2, \quad h(a) = \frac{a^2}{l^2} + 1 - \frac{16\pi GM}{3V_3 a^2}. \quad (11)$$

Here V_3 is the volume of the unit 3 sphere. If one chooses new coordinates (z, τ) by

$$\begin{aligned} \frac{e^{2A}}{h(a)} A_{,z}^2 - h(a) t_{,z}^2 &= 1, & \frac{e^{2A}}{h(a)} A_{,z} A_{,\tau} - h(a) t_{,z} t_{,\tau} &= 0 \\ \frac{e^{2A}}{h(a)} A_{,\tau}^2 - h(a) t_{,\tau}^2 &= -e^{2A}. \end{aligned} \quad (12)$$

the metric takes the warped form (10). Here $a = le^A$. In general we might be unable to rewrite globally the metric in (11) in the form of (10). Nevertheless, it can be done in the neighbourhood of the brane, what is necessary here. Further choosing a coordinate \tilde{t} by $d\tilde{t} = le^A d\tau$, the metric on the brane takes FRW form:

$$e^{2A} \tilde{g}_{\mu\nu} dx^\mu dx^\nu = -d\tilde{t}^2 + l^2 e^{2A} d\Omega_3^2. \quad (13)$$

By solving Eqs.(12), we have

$$H^2 = A_{,z}^2 - h e^{-2A} = A_{,z}^2 - \frac{1}{l^2} - \frac{1}{a^2} + \frac{16\pi GM}{3V_3 a^4}. \quad (14)$$

Here the Hubble constant H is introduced: $H = \frac{dA}{d\tilde{t}}$. On the other hand, from (9) one gets

$$\begin{aligned} A_{,z} = & \frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left((H_{\tilde{t}\tilde{t}\tilde{t}} + 4H_{\tilde{t}}^2 + 7HH_{\tilde{t}\tilde{t}} + 18H^2H_{\tilde{t}} + 6H^4) \right. \right. \\ & + \frac{4}{a^2} (H_{\tilde{t}} + H^2) \Big) + 4(b+b') \left((H_{\tilde{t}\tilde{t}\tilde{t}} + 4H_{\tilde{t}}^2 \right. \\ & \left. \left. + 7HH_{\tilde{t}\tilde{t}} + 12H^2H_{\tilde{t}}) - \frac{2}{a^2} (H_{\tilde{t}} + H^2) \right) \right\}. \end{aligned} \quad (15)$$

Then combining (14) and (15), we find

$$\begin{aligned} H^2 = & -\frac{1}{l^2} - \frac{1}{a^2} + \frac{16\pi GM}{3V_3 a^4} + \left[\frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left((H_{\tilde{t}\tilde{t}\tilde{t}} + 4H_{\tilde{t}}^2 \right. \right. \right. \\ & + 7HH_{\tilde{t}\tilde{t}} + 18H^2H_{\tilde{t}} + 6H^4) + \frac{4}{a^2} (H_{\tilde{t}} + H^2) \Big) \\ & \left. \left. + 4(b+b') \left((H_{\tilde{t}\tilde{t}\tilde{t}} + 4H_{\tilde{t}}^2 + 7HH_{\tilde{t}\tilde{t}} + 12H^2H_{\tilde{t}}) - \frac{2}{a^2} (H_{\tilde{t}} + H^2) \right) \right\} \right]^2. \end{aligned} \quad (16)$$

This expresses the quantum correction to the corresponding brane equation in [9]. In fact, if we put $b = b' = 0$, Eq.(16) reduces to the classical FRW equation

$$H^2 = -\frac{1}{a^2} + \frac{16\pi GM}{3V_3 a^4}. \quad (17)$$

Further by differentiating Eq.(16) with respect to \tilde{t} , we obtain

$$H_{,\tilde{t}} = \frac{1}{a^2} - \frac{32\pi GM}{3V_3 a^4} + \frac{\pi G}{3H} \left[\frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left((H_{\tilde{t}\tilde{t}\tilde{t}} + 4H_{\tilde{t}}^2 \right. \right. \right.$$

$$\begin{aligned}
& +7HH_{,\bar{t}\bar{t}} + 18H^2H_{,\bar{t}} + 6H^4) + \frac{4}{a^2} (H_{,\bar{t}} + H^2)) \\
& +4(b+b') \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 + 7HH_{,\bar{t}\bar{t}} + 12H^2H_{,\bar{t}}) - \frac{2}{a^2} (H_{,\bar{t}} + H^2) \right) \Big] \\
& \times \left\{ -4b' \left((H_{,\bar{t}\bar{t}\bar{t}\bar{t}} + 15H_{,\bar{t}}H_{,\bar{t}\bar{t}} + 7HH_{,\bar{t}\bar{t}\bar{t}} + 18H^2H_{,\bar{t}\bar{t}} \right. \right. \\
& \left. \left. + 36HH_{,\bar{t}}^2 + 24H^3H_{,\bar{t}}) + \frac{4}{a^2} (H_{,\bar{t}\bar{t}} - 2H^3) \right) \right. \\
& \left. +4(b+b') \left((H_{,\bar{t}\bar{t}\bar{t}\bar{t}} + 15H_{,\bar{t}}H_{,\bar{t}\bar{t}} + 7HH_{,\bar{t}\bar{t}\bar{t}} + 12H^2H_{,\bar{t}\bar{t}} \right. \right. \\
& \left. \left. + 24HH_{,\bar{t}}^2) - \frac{2}{a^2} (H_{,\bar{t}\bar{t}} - 2H^2) \right) \right\} . \tag{18}
\end{aligned}$$

One can rewrite the above equations (16) and (18) in the form of FRW equations:

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G_4 \rho}{3} \tag{19}$$

$$\begin{aligned}
\rho = \frac{l}{a} \left[\frac{M}{V_3 a^3} + \frac{3a}{16\pi G} \left[\left[\frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 + 7HH_{,\bar{t}\bar{t}} \right. \right. \right. \right. \right. \right. \\
\left. \left. \left. + 18H^2H_{,\bar{t}} + 6H^4) + \frac{4}{a^2} (H_{,\bar{t}} + H^2) \right) + 4(b+b') \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 \right. \right. \right. \right. \right. \\
\left. \left. \left. + 7HH_{,\bar{t}\bar{t}} + 12H^2H_{,\bar{t}}) - \frac{2}{a^2} (H_{,\bar{t}} + H^2) \right) \right\} \right]^2 - \frac{1}{l^2} \right] , \tag{20}
\end{aligned}$$

$$H_{,\bar{t}} = \frac{1}{a^2} - 4\pi G_4(\rho + p) \tag{21}$$

$$\begin{aligned}
\rho + p = \frac{l}{a} \left[\frac{4M}{3V_3 a^3} - \frac{1}{24l^3 H} \left[\frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 + 7HH_{,\bar{t}\bar{t}} \right. \right. \right. \right. \right. \right. \\
\left. \left. \left. + 18H^2H_{,\bar{t}} + 6H^4) + \frac{4}{a^2} (H_{,\bar{t}} + H^2) \right) + 4(b+b') \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 + 7HH_{,\bar{t}\bar{t}} + 12H^2H_{,\bar{t}}) \right. \right. \right. \\
\left. \left. \left. - \frac{2}{a^2} (H_{,\bar{t}} + H^2) \right) \right\} \right] \\
\times \left\{ -4b' \left((H_{,\bar{t}\bar{t}\bar{t}\bar{t}} + 15H_{,\bar{t}}H_{,\bar{t}\bar{t}} + 7HH_{,\bar{t}\bar{t}\bar{t}} + 18H^2H_{,\bar{t}\bar{t}} + 36HH_{,\bar{t}}^2 \right. \right. \\
\left. \left. + 24H^3H_{,\bar{t}}) + \frac{4}{a^2} (H_{,\bar{t}\bar{t}} - 2H^3) \right) + 4(b+b') \left((H_{,\bar{t}\bar{t}\bar{t}\bar{t}} + 15H_{,\bar{t}}H_{,\bar{t}\bar{t}} \right. \right. \\
\left. \left. + 7HH_{,\bar{t}\bar{t}\bar{t}} + 12H^2H_{,\bar{t}\bar{t}} + 24HH_{,\bar{t}}^2) - \frac{2}{a^2} (H_{,\bar{t}\bar{t}} - 2H^2) \right) \right\} . \tag{22}
\end{aligned}$$

Here 4d Newton constant G_4 is given by

$$G_4 = \frac{2G}{l} . \quad (23)$$

and quantum corrections from CFT are included into the definition of energy (pressure). These quantum corrected FRW equations are written from quantum-induced brane-world perspective. Similar equations from the point of view of 4d brane observer (who does not know about 5d AdS bulk) have been presented in ref.[7]. Clearly, brane-world approach gives more information. As the correction terms include higher derivatives, these terms become relevant when the universe changes its size very rapidly as in the very early universe.

It is not so clear if the energy density ρ and the pressure p satisfy the energy conditions from the expressions in (20) and (22), because quantum effects generally may violate the energy conditions. For the solution of (19), however, ρ is always positive since (19) can be rewritten

$$\rho = \frac{3}{8\pi G_4} \left(H^2 + \frac{1}{a^2} \right) > 0. \quad (24)$$

We also have from (21)

$$\rho + p = \frac{1}{4\pi G_4} \left(\frac{1}{a^2} - H_{,\tilde{t}} \right) . \quad (25)$$

Therefore the weak energy condition should be satisfied if $\frac{1}{a^2} - H_{,\tilde{t}} > 0$ in the solution. In order to clarify the situation, we consider the specific case of $b + b' = 0$ as in $\mathcal{N} = 4$ theory and we assume that b' is small. Then from (19) and (21) and by differentiating (21) with respect \tilde{t} , one gets

$$\begin{aligned} H^2 &= -\frac{1}{a^2} + \frac{8\pi G_4 M l}{3V_3 a^4} + \mathcal{O}(b') , \quad H_{,\tilde{t}} = \frac{1}{a^2} - \frac{16\pi G_4 M l}{3V_3 a^4} + \mathcal{O}(b') , \\ H_{,\tilde{t}\tilde{t}} &= -\frac{2}{a^2} H + \frac{64\pi G_4 M l}{3V_3 a^4} H + \mathcal{O}(b') , \quad \text{etc.} \end{aligned} \quad (26)$$

Then by using (20) and (22), we find

$$\begin{aligned} \rho &= \frac{M l}{V_3 a^4} - \frac{b'}{2} \left(\frac{8\pi G_4 M l}{V_3 a^6} - \frac{128\pi^2 G_4^2 M^2 l^2}{3V_3^2 a^8} \right) + \mathcal{O}(b'^2) , \\ p &= \frac{M l}{3V_3 a^4} - \frac{b'}{2} \left(\frac{8\pi G_4 M l}{V_3 a^6} - \frac{640\pi^2 G_4^2 M^2 l^2}{9V_3^2 a^8} \right) + \mathcal{O}(b'^2) . \end{aligned} \quad (27)$$

The correction part of ρ is not always positive but ρ itself should be positive, what is clear from (24). One also gets

$$\rho + p = \frac{4Ml}{3V_3a^4} - \frac{b'}{2} \left(\frac{16\pi G_4 M l}{V_3 a^6} - \frac{1024\pi^2 G_4^2 M^2 l^2}{9V_3^2 a^8} \right) + \mathcal{O}(b'^2) . \quad (28)$$

Then the correction part seems to be not always positive and the weak energy condition might be broken. As the above discussion is based on the perturbation theory, we will discuss the weak energy condition later using the de Sitter type brane universe solution.

Let us consider the solution of quantum-corrected FRW equation (16). Assume the de Sitter type solution

$$a = A \cosh B\tilde{t} . \quad (29)$$

Substituting (29) into (16), one finds the following equations should be satisfied:

$$0 = -\frac{1}{B^2} - \frac{1}{l^2} + \left(\frac{1}{l} - 8\pi G b' B^4 \right)^2 \quad (30)$$

$$0 = B^2 - \frac{1}{A^2} + 2 \left(\frac{1}{l} - 8\pi G b' B^4 \right) \frac{\pi G}{3} (24b' + 8b) \left(B^4 - \frac{B^2}{A^2} \right) \quad (31)$$

$$0 = \frac{16\pi G M}{3V_3} + \left(\frac{\pi G}{3} \right)^2 (24b' + 8b)^2 \left(B^4 - \frac{B^2}{A^2} \right)^2 . \quad (32)$$

Eq.(30) tells that there is no de Sitter type solution if there is no quantum correction, or if $b' = 0$. Eq.(32) tells that if the black hole mass M is non-vanishing and positive, there is no any solution of the de Sitter-like brane. When $M = 0$, Eqs.(31) and (32) are trivially satisfied if $A^2 = \frac{1}{B^2}$. Actually this case corresponds to well-known anomaly-driven inflation [18] (for recent discussion, see [19]). Eq.(30) has unique non-trivial solution for B^2 , which corresponds to the de Sitter brane universe in [11, 12].

When $M < 0$, there is no horizon and the curvature singularity becomes naked. We will, however, formally consider the case since there is no de Sitter-like brane solution in the classical case ($b' = 0$) even if M is negative. If $M \neq 0$ or $A^2 \neq \frac{1}{B^2}$, Eq.(31) has the following form:

$$0 = 1 + 2 \left(\frac{1}{l} - \frac{8\pi G b'}{l^4} B^4 \right) \frac{\pi G}{3l^4} (24b' + 8b) B^2 . \quad (33)$$

Eq.(33) is not always compatible with Eq.(30) and gives a non-trivial constraint on G , l , b and b' . If the constraint is satisfied, B^2 can be uniquely determined by (30) or (33). Then (32) can be solved with respect to A^2 .

Now we consider the above constraint and solution for B^2 . By combining (30) and (33), one obtains

$$0 = B^6 + \frac{1}{l^2}B^4 - \frac{1}{\eta}, \quad \eta \equiv 4(24b' + 8b) \left(\frac{\pi G}{3} \right)^2 \quad (34)$$

$$0 = \left(\frac{1}{l^2} + B^2 \right)^3 - \left\{ \frac{1}{l} \left(\frac{1}{l^2} + B^2 \right) - \zeta \right\}, \quad \zeta \equiv \frac{6b'}{24b' + 8b} \left(\frac{3}{\pi G} \right). \quad (35)$$

In most of cases, η is negative and ζ is positive. The explicit solution of (34) is given by

$$B^2 = -\frac{1}{3l^2} + \left(\frac{1}{27l^6} - \frac{1}{2\eta} + \sqrt{\frac{1}{4\eta^2} - \frac{1}{27l^6\eta}} \right)^{\frac{1}{3}} + \left(\frac{1}{27l^6} - \frac{1}{2\eta} - \sqrt{\frac{1}{4\eta^2} - \frac{1}{27l^6\eta}} \right)^{\frac{1}{3}}. \quad (36)$$

On the other hand, if $\frac{\zeta^4}{4} - \frac{\zeta^3}{27l^4} > 0$, the solution of (35) is given by

$$B^2 = -\frac{2}{3l^2} + \left(-\frac{1}{27l^6} + \frac{\zeta}{3l^3} - \frac{\zeta^2}{2} + \sqrt{\frac{\zeta^4}{4} - \frac{\zeta^3}{27l^4}} \right)^{\frac{1}{3}} + \left(-\frac{1}{27l^6} + \frac{\zeta}{3l^3} - \frac{\zeta^2}{2} - \sqrt{\frac{\zeta^4}{4} - \frac{\zeta^3}{27l^4}} \right)^{\frac{1}{3}}. \quad (37)$$

or if $\frac{\zeta^4}{4} - \frac{\zeta^3}{27l^4} < 0$, the solutions are

$$B^2 + \frac{2}{3l^2} = \xi + \xi^*, \quad \xi\omega + \xi^*\omega^2, \quad \xi\omega^2 + \xi^*\omega. \quad (38)$$

Here

$$\xi = \left(-\frac{1}{27l^6} + \frac{\zeta}{3l^3} - \frac{\zeta^2}{2} + i\sqrt{\frac{\zeta^3}{27l^4} - \frac{\zeta^4}{4}} \right)^{\frac{1}{3}}, \quad \omega = e^{\frac{2i\pi}{3}}. \quad (39)$$

Then if the solution (36) coincides with any of the solutions (37) or (38), there occurs quantum-induced de Sitter-like brane realized in d5 AdS BH. In a sense, we got the extension of scenario of refs.[11, 12] for quantum-induced brane-worlds within AdS/CFT set-up when bulk is given by d5 AdS BH.

For the de Sitter type solution (29), Eq.(25) has the following form:

$$\rho + p = \frac{1}{4\pi G_4} \left(\frac{1}{A^2} - B^2 \right) \frac{1}{\cosh^2 B\tilde{t}} . \quad (40)$$

Then the weak energy condition can be satisfied if

$$\frac{1}{A^2} \geq B^2 . \quad (41)$$

For the exact de Sitter solution corresponding to $M = 0$, we have $\frac{1}{A^2} = B^2$ and Eq.(41) is satisfied. For more general solution in (36) or (38), B and A non-trivially depend on the parameters G , M , b and b' and it is not so clear if Eq.(41) is always satisfied.

Since the solution whose form is $a = A \cosh B\tilde{t}$ exists when the parameters satisfy a special constraint and the black hole mass M is negative, we now consider a perturbation from the de Sitter brane solution by assuming that the black hole mass M is small. By choosing a as

$$\ln a = \ln \left(\frac{1}{B} \cosh B\tilde{t} \right) + h(\tilde{t}) , \quad (42)$$

we expand (16) in the first order of M and h :

$$\begin{aligned} 0 = & -2B \tanh B\tilde{t} h_{,\tilde{t}} + 2B^2 \cosh^2 B\tilde{t} + \frac{16\pi G M B^4}{3V_3 \cosh^4 B\tilde{t}} \\ & + \frac{2\pi G}{3} \left(\frac{1}{l} - 8\pi G B^4 b' \right) \left\{ -4b' \left(h_{,\tilde{t}\tilde{t}\tilde{t}\tilde{t}} + 7B \tanh B\tilde{t} h_{,\tilde{t}\tilde{t}\tilde{t}} \right. \right. \\ & + B^2 \left(18 - \frac{6}{\cosh^2 B\tilde{t}} \right) h_{,\tilde{t}\tilde{t}} + B^3 \left(\frac{24 \sinh B\tilde{t}}{\cosh B\tilde{t}} + \frac{12 \sinh B\tilde{t}}{\cosh^3 B\tilde{t}} \right) h_{,\tilde{t}} \\ & \left. \left. - \frac{8B^4}{\cosh^2 B\tilde{t}} h \right) + 4(b + b') \left(h_{,\tilde{t}\tilde{t}\tilde{t}} + 7B \tanh B\tilde{t} h_{,\tilde{t}\tilde{t}} \right. \right. \\ & \left. \left. + B^2 \left(12 - \frac{2}{\cosh^2 B\tilde{t}} \right) h_{,\tilde{t}\tilde{t}} + \frac{14B^3 \sinh B\tilde{t}}{\cosh^3 B\tilde{t}} h_{,\tilde{t}} - \frac{4B^4}{\cosh^2 B\tilde{t}} h \right) \right\} \quad (43) \end{aligned}$$

When \tilde{t} is small, the solution of (43) can be given by the power series of \tilde{t} as $h(\tilde{t}) = \sum_{n=0}^{\infty} h_n \tilde{t}^n$ with a proper boundary condition. On the other hand,

when \tilde{t} is large, Eq.(43) has the following form:

$$\begin{aligned}
0 = & -2Bh_{,\tilde{t}} + \frac{16\pi GMB^4}{3V_3}e^{-4B\tilde{t}} \\
& + \frac{2\pi G}{3} \left(\frac{1}{l} - 8\pi GB^4b' \right) \left\{ -4b' \left(h_{,\tilde{t}\tilde{t}\tilde{t}\tilde{t}} + 7B\tilde{t}h_{,\tilde{t}\tilde{t}\tilde{t}} + 18B^4h_{,\tilde{t}\tilde{t}} + 24B^3h_{,\tilde{t}} \right) \right. \\
& \left. + 4(b+b') \left(h_{,\tilde{t}\tilde{t}\tilde{t}} + 7B\tilde{t}h_{,\tilde{t}\tilde{t}} + 12B^2h_{,\tilde{t}} \right) \right\} .
\end{aligned} \tag{44}$$

The solution of (44) is given by

$$\begin{aligned}
h &= h_0 e^{-4B\tilde{t}} + M \text{ independent terms} \\
h_0 &\equiv \frac{\frac{16\pi GM}{3V_3}}{\frac{800\pi Gb}{3} \left(\frac{1}{l} - 8\pi GB^4b' \right) - \frac{8}{B^3}} .
\end{aligned} \tag{45}$$

The M independent terms vanish if we impose a condition that h vanishes when $E = 0$. As the solution seems to exist consistently, there will be (approximate) de Sitter like solutions for a range of the parameters. Therefore the quantum correction seems to induce the de Sitter like brane not only in case of $M = 0$ but even in the case of $M \neq 0$.

For the above obtained perturbative solution (42), Eq.(25) has the following form:

$$\rho + p = -\frac{1}{4\pi G_4} \frac{1}{h_{,\tilde{t}\tilde{t}}} . \tag{46}$$

If we take M -independent terms in (45) to vanish, (46) has the following form:

$$\rho + p = -\frac{1}{4\pi G_4} \frac{1}{16B^2h_0} . \tag{47}$$

By using the expression of h_0 in (45), the weak energy condition is satisfied, at least for large \tilde{t} , if

$$\frac{800\pi Gb}{3} \left(\frac{1}{l} - 8\pi GB^4b' \right) - \frac{8}{B^3} < 0 . \tag{48}$$

This depends on the field contents on the brane.

3 Properties of inflationary brane in AdS-Schwarzschild Black Hole

Let us briefly discuss the properties of the found inflationary brane universe. When a is large, the metric (11) has the following form:

$$ds_{\text{AdS-S}}^2 \rightarrow \frac{a^2}{l^2} \left(dt^2 + l^2 d\Omega_3^2 \right) , \quad (49)$$

which tells that the CFT time t_{CFT} is equal to the AdS time t times the factor $\frac{a}{l}$, at least when the radius of the brane is large enough:

$$t_{\text{CFT}} = \frac{a}{l} t . \quad (50)$$

Therefore the energy E_{CFT} in CFT is related with the energy E_{AdS} in AdS by [9]

$$E_{\text{CFT}} = \frac{l}{a} E_{\text{AdS}} . \quad (51)$$

The factor $\frac{l}{a}$ in front of Eqs.(20) and (22) appears due to the above scaling of the energy in (51) or time in (50).

The AdS₅-Schwarzschild black hole solution in (11) has a horizon at $a = a_H$, where $h(a)$ vanishes [16]:

$$h(a_H) = \frac{a_H^2}{l^2} + 1 - \frac{16\pi G M}{3V_3 a_H^2} = 0 . \quad (52)$$

Then considering the moment the brane crosses these points and using (16), one gets

$$\begin{aligned} H = \pm \left[\frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 + 7HH_{,\bar{t}\bar{t}} + 18H^2H_{,\bar{t}} + 6H^4) \right. \right. \right. \\ \left. \left. + \frac{4}{a_H^2} (H_{,\bar{t}} + H^2) \right) + 4(b + b') \left((H_{,\bar{t}\bar{t}\bar{t}} + 4H_{,\bar{t}}^2 \right. \right. \\ \left. \left. + 7HH_{,\bar{t}\bar{t}} + 12H^2H_{,\bar{t}}) - \frac{2}{a_H^2} (H_{,\bar{t}} + H^2) \right) \right\} \right] . \quad (53) \end{aligned}$$

The sign \pm depends on whether the brane is expanding or contracting. Obviously, if the higher derivative of the Hubble constant H is large, the quantum correction becomes essential.

We now assume that the brane behaves as de Sitter (inflationary) space in (29) near the horizon. As it has been shown, this is quantum-induced brane Universe. (Parameters B , A are defined by quantum effects). Note that this is not the exact solution for positive (non-vanishing) black hole mass $M > 0$ while there can be another kind of solution, not of de Sitter type. The above assumption, however, is not so unnatural since it only means that the brane universe expands (or shrinks) uniformly near the horizon. Then Eqs.(52) and (53) have the following forms:

$$h(a_H) = \frac{A^2 \cosh^2 \tilde{t}_H}{l^2} + 1 - \frac{16\pi GM}{3V_3 A^2 \cosh^2 \tilde{t}_H} = 0 \quad (54)$$

$$H = \pm \left[\frac{1}{l} + \frac{\pi G}{3} \left\{ -4b' \left(-4 \left(B^4 - \frac{B^2}{A^2} \right) \frac{1}{\cosh^2 \tilde{t}_H} + 6B^4 \right) + 8(b + b') \left(B^4 - \frac{B^2}{A^2} \right) \frac{1}{\cosh^2 \tilde{t}_H} \right\} \right]. \quad (55)$$

Here the brane crosses the horizon when $\tilde{t} = \tilde{t}_H$. Thus, quantum-corrected Hubble parameter at the horizon is defined. The quantum correction becomes large when the rate B of expansion of the universe is large.

Let the entropy \mathcal{S} of CFT on the brane is given by the Bekenstein-Hawking entropy of the AdS₅ black hole $\mathcal{S} = \frac{V_H}{4G}$. Here V_H is the area of the horizon, which is equal to the spatial brane when the brane crosses the horizon: $V_H = a_H^3 V_3$. If the total entropy \mathcal{S} is constant during the cosmological evolution, the entropy density s is given by (see [9])

$$s = \frac{\mathcal{S}}{a^3 V_3} = \frac{l a_H^3}{2G_4 a^3}. \quad (56)$$

Here Eq.(23) is used. The expression in (56) is identical with the classical one. The quantum correction appears when we express s in terms of the quantities in brane universe, say H , $H_{\tilde{t}}$ etc., by using (53).

The Hawking temperature of the black hole is given by (see [9])

$$T_H = \frac{h'(a_H)}{4\pi} = \frac{a_H}{2\pi l^2} + \frac{8GM}{3V_3 a_H^3} = \frac{a_H}{\pi l^2} + \frac{1}{2\pi a_H}. \quad (57)$$

Here (52) is used. As in (50), the temperature T on the brane is different from that of AdS₅ by the factor $\frac{l}{a}$:

$$T = \frac{l}{a} T_H = \frac{a_H}{\pi a l} + \frac{l}{2\pi a a_H}. \quad (58)$$

Especially when $a = a_H$

$$T = \frac{1}{\pi l} + \frac{l}{2\pi a_H^2} . \quad (59)$$

Then from (18) and (53), one gets

$$\begin{aligned} T = & \frac{l}{\pi} \left[-2H_{,\tilde{t}} \pm \frac{\pi G}{3} \left\{ -4b' \left((H_{,\tilde{t}\tilde{t}\tilde{t}} + 15H_{,\tilde{t}}H_{,\tilde{t}\tilde{t}} + 7HH_{,\tilde{t}\tilde{t}} \right. \right. \right. \\ & + 18H^2H_{,\tilde{t}\tilde{t}} + 36HH_{,\tilde{t}}^2 + 24H^3H_{,\tilde{t}}) + \frac{4}{a^2} (H_{,\tilde{t}\tilde{t}} - 2H^3) \Big) \\ & + 4(b + b') \left((H_{,\tilde{t}\tilde{t}\tilde{t}} + 15H_{,\tilde{t}}H_{,\tilde{t}\tilde{t}} + 7HH_{,\tilde{t}\tilde{t}} + 12H^2H_{,\tilde{t}\tilde{t}} \right. \\ & \left. \left. + 24HH_{,\tilde{t}}^2) - \frac{2}{a_H^2} (H_{,\tilde{t}\tilde{t}} - 2H^2) \right) \right\} \Big] . \quad (60) \end{aligned}$$

Again, if the higher derivative of the Hubble constant H is large, the quantum correction becomes important. For the solution (29), we have

$$T = \frac{l}{\pi} \left[-2H_{,\tilde{t}} \pm \frac{\pi G}{3} (48b' + 16b) \left(B^4 - \frac{B^2}{A^2} \right) \frac{\sinh B\tilde{t}_H}{\cosh^3 B\tilde{t}_H} \right] . \quad (61)$$

The quantum correction becomes dominant when $B\tilde{t}_H$ is of order unity but B ($\neq \frac{1}{A}$) is large or A is small. Since the radius of the horizon is given by $a_H = A \cosh B\tilde{t}_H$, this might mean that if quantum correction is large then the radius of the black hole is small. Eq.(58) tells that the temperature on the brane should be determined by the Hawking temperature of the black hole. If there is no quantum correction, the temperature is directly related with $H_{,\tilde{t}}$ but Eq.(60) or (61) tells that the quantum correction breaks this simple relation. Thus, the main qualitative role of quantum effects was to provide the explicit inflationary brane universe solution which does not exist otherwise (at least, in our approach to brane-worlds). The parameters of quantum CFT enter to the Hubble constant, Hawking temperature, energy (entropy) and they may change their classical values.

4 Discussion

In summary, it is shown that in d5 AdS black hole background, the inflationary brane induced by CFT quantum effects may occur in the same way

as in refs.[11, 12]. It is important to note that brane stress tensor is completely defined by dual quantum CFT (and also probably, by brane QG) and it is not chosen by hands as it happens often in the traditional brane-world scenarios, where the brane tension is fine-tuned. We also investigated the energy conditions and found that the energy density is always positive but the weak energy condition might be broken by the quantum effects. When the quantum-induced brane crosses horizon of AdS BH, the Hubble constant, brane entropy and the Hawking temperature (also at the horizon) are found with account of quantum corrections. The similarity between CFT entropy at the horizon and FRW equations discovered in refs.[4, 9] is extended for the presence of quantum effects. These results may be important for the generalization of cosmological entropy bounds in the case of quantum gravity. From another side, it would be interesting to use such study with the purpose of extension of AdS/CFT correspondence for cosmological (AdS) backgrounds [17]. Clearly, there are many questions about physical interpretation of some of the obtained results which should be carefully investigated in the future.

Acknowledgments The work by SDO has been supported in part by CONACyT (CP, Ref.990356 and grant 28454E).

References

- [1] S.W. Hawking, *Comm.Math.Phys.* **43** (1974) 199.
- [2] G.'t Hooft, gr-qc/9312026; L. Susskind, *J.Math.Phys.* **36** (1995) 6337; W. Fischler and L. Susskind, hep-th/9806039; R. Easter and D. Lowe, hep-th/9902088, *Phys.Rev.Lett.* **82** (1999) 4967; D. Bak and S. Rey, hep-th/9902173, *Class.Quant.Grav.* **17** (2000) L83; N. Kaloper and A. Linde, hep-th/9904120; R. Bousso, hep-th/9905177, *JHEP* **9907** (1999) 004; B. Wang, E. Abdalla and T. Osada, *Phys.Rev.Lett.* **85** (2000) 5507.
- [3] S.W. Hawking, J. Maldacena and A. Strominger, hep-th/0002145.
- [4] E. Verlinde, hep-th/0008140.
- [5] J.M. Maldacena, *Adv.Theor.Math.Phys.* **2** (1998) 231; E. Witten, *Adv.Theor.Math.Phys.* **2** (1998) 253; S. Gubser, I. Klebanov and A. Polyakov, *Phys.Lett.* **B428** (1998) 105;

- [6] D. Kutasov and F. Larsen, hep-th/0009244; F.-L. Lin, hep-th/0010127; B. Wang, E. Abdalla and R.-K. Su, hep-th/0101073; Y. S. Myung, hep-th/0102184; R.-G. Cai, hep-th/0102113; R. Brustein, S. Foffa and G. Veneziano, hep-th/0101083; A. Biswas and S. Mukherji, hep-th/0102138.
- [7] S. Nojiri and S.D. Odintsov, hep-th/0011115.
- [8] S.S. Gubser, *Phys.Rev.* **D63** (2001) 084017, hep-th/9912001.
- [9] I. Savonije and E. Verlinde, hep-th/0102042.
- [10] L. Randall and R. Sundrum, *Phys.Rev.Lett.* **83** (1999) 3370, hep-th/9905221; *Phys.Rev.Lett.* **83** (1999) 4690, hep-th/9906064.
- [11] S.W. Hawking, T. Hertog and H.S. Reall, hep-th/0003052, *Phys.Rev.* **D62** (2000) 043501.
- [12] S. Nojiri, S.D. Odintsov and S. Zerbini, hep-th/0001192, *Phys.Rev.* **D62** (2000) 064006; S. Nojiri and S.D. Odintsov, hep-th/0004097, *Phys.Lett.* **B484** (2000) 119.
- [13] L. Anchordoqui, C. Nunez and K. Olsen, hep-th/0007064; K. Koyama and J. Soda, hep-th/0101164; T. Shiromizu and D. Ida, hep-th/0102035.
- [14] S. Nojiri, O. Obregon and S.D. Odintsov, hep-th/0005127, *Phys.Rev.* **D62** (2000) 104003; S. Nojiri, O. Obregon, S.D. Odintsov and V.I. Tkach, hep-th/0101003; S. Nojiri and S.D. Odintsov, hep-th/0102032.
- [15] S. Nojiri, S.D. Odintsov and S. Zerbini, hep-th/0006115, *Class.Quant.Grav.* **17** (2000) 4855.
- [16] S. Hawking and D. Page, *Comm.Math.Phys.* **87** (1983) 577.
- [17] S. Nojiri and S.D. Odintsov, hep-th/0008160, *Phys.Lett.* **B494** (2000) 135.
- [18] A. Starobinsky, *Phys.Lett.* **B91** (1980) 99.
- [19] S.W. Hawking, T. Hertog and H.S. Reall, hep-th/0010252; K. Hamada, hep-th/0101100.